NEURAL NETWORK SIMULATION OF δ-CORRELATED STOCHASTIC SIGNALS

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Abstract. Artificial neural networks (ANNs) are widely used as "black-box" models of different complex processes and systems. Although the neural network simulation of deterministic processes is a well-known area, ANN simulation of stochastic ones is still at the frontier of the ANN methodology. In the current work, the method of ANN simulation of δ-correlated stochastic signals is proposed. The main advantage of the scheme is the utilization of a standard multilayer perceptron instead of complex stochastic ANN structures. The network receives input parameters of simulation together with basic random values and generates the desired stochastic signal.

1. Introduction

The method of computer simulation has been proved as a powerful tool for the exploration of different complex processes and systems. They gain a considerable attention in recent years, when being used for adequate forecasting of the behaviour of explored systems under different external or internal conditions. Classical approximation methods are generally used for the analysis of well-known analytical expressions, which are far too simple to describe the real physical processes. For the correct interpretation of the experimental data computer simulation must be included in the process of data analysis. One of the forms of such application is simulation-based fitting (SBF) [1, 2]. The idea of SBF is the approximation of experimental data by synthetic data obtained *via* simulation modelling. In comparison to standard analytical data fitting techniques, SBF has the advantage that it fits natural physical parameters of the system itself and gives a direct insight in how they affect the experimental characteristics of the system.

However, it is not always necessary to operate with a simulation model (or a "white box" model), which gives precise results but is far more computationally expensive than analytical approximation. For example, in SBF only parameters of the model are modified, while its structure holds constant. In such a case, it may be useful to perform a "black box" modelling, which still operates with real physical parameters, but can be performed much faster. It was proposed to use artificial neural networks (ANNs) [3] as "black box" simulators of physical processes. ANNs are known as universal approximators and classifiers, and are characterised as noise-stable data processing tools with the generalisation ability. The neural network approximation of a process or its simulation model allows a significant decrease of fitting time and the noise uncertainty during utilisation of SBF [1]. Although the ANN simulation of deterministic processes is well known and studied, the simulation of stochastic ones is still at the frontier of the ANN methodology. This paper is devoted to neural network simulation of δ-correlated stochastic signals.

2. General approach to ANN simulation of a stochastic signal

To simulate stochastic processes two approaches can be applied. The first one – is to use a stochastic ANN (Boltzmann machine, etc.). However, this area of ANN is not completely studied yet and there are unsolved problems concerning network structure determination and training procedure. Another approach, proposed in the current paper, is application of a standard feed-forward network, like a multilayer perceptron [3], with slight modifications. To utilize a deterministic network for generation of a random signal one should put into it a source of randomness. To do so the random signal should be given into ANN inputs. In fact, such network operates as an abstract function that transfers the set of uniform random values $\{x_i\}\in\mathbf{R}$ and deterministic simulations parameters p to arbitrary distributed $\{y_j\}\in\mathbf{Y}$. This general approach is demonstrated in fig. 1. The simulation parameters, together with uniformly distributed random values are given to the network, and the simulated random value is taken from its output.

3. Training algorithm

It should be noted, that standard "back-propagation error" methods are not applicable to train ANN in this case. The general scheme used in the work is demonstrated in fig. 2. Each training pair is presented by a vector of input simulation parameters p (block 1) and a sufficiently long output random signal $y(t)$ (block 3) obtained from the experimental system (block 2). The statistical values are calculated from $y(t)$: mean, standard deviation, estimation of probability density, minimal and maximal values (block 4). During training, parameters p together with a set of random vectors $x(t)\in\mathbf{R}$ (block 5) are given into the ANN, which produces a sufficiently long random vector $y^*(t)$ (block 8). The same statistical values are calculated for it. The weighted distinction of these values for $y^*(t)$ and $y(t)$ gives the error of ANN simulation. The ANN weight coefficients can be modified iteratively using one of standard stochastic training algorithm (Monte Carlo or genetic algorithm).

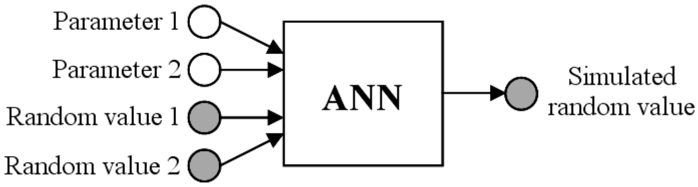


Fig. 1. General scheme of neural network simulation of a stochastic signal

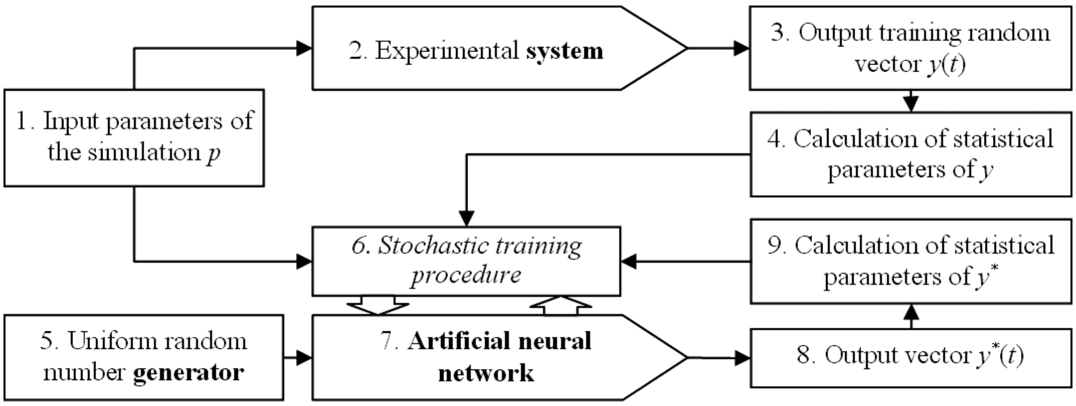


Fig. 2. Training of the ANN for the approximation of stochastic signal

4. Simulation of the test signal

To test this methodology the random signal with a normal distribution (see eq. 1) was successfully generated by a 3 layer perceptron with 8x8x1 nonlinear (hyperbolic tangent) neurons.

$$y(t) = p_1 \cdot n(t) + p_2, \quad (1)$$

where p_1, p_2 – "simulation" parameters from rangers [0,1] and [-1,1] respectively, $n(t)$ – Gaussian stochastic signal with m=0, σ=1. Two uniformly distributed random signals were taken as $\{x_i\}$.

The statistical tests were performed for the case of p_1=0.5, p_2=1. The normality of the generated vector (4096 elements) was tested *via* Kolmogorov-Smirnov criterion. The results are presented in the table 1. The test confirms that the stochastic signal simulated by ANN has the distribution very close to the normal with m=1, σ=0.5. However, we should remember that this is not the signal (1), but its approximation.

Table 1. Results of the statistical test of the stochastic signal generated by ANN

Description	Simulated	Theoretical
mean	0.9914	1
mean estimates for 95% confidence	0.9755÷1.0072	–
standard deviation	0.5167	0.5
standard deviation estimates for 95% confidence	0.5057÷0.5281	–
λ, critical value for Kolmogorov-Smirnov criterion	0.0834	1.358
conclusion of Kolmogorov-Smirnov criterion with the desired significance level of 0.05	$\lambda_{sim}<\lambda_{theor}$ => the null hypothesis is not rejected, the distribution can be considered as a normal one	

5. Simulation of fluctuating transition in a noise generator

Experimental data. Consider the experimental system in which the stochastic process occurs – a semiconductor noise generator. Most of the semiconductor devices (diodes and triodes) work in dynamic non-equilibrium conditions. Noises in these devices have a stochastic nature and limit their sensitivity to weak signals; therefore, in most cases the noises are the object of reduction and optimization. However, noises of semiconductor devices can be utilized themselves, for instance, as a part of physical random number generators. These factors impel to conduct specific researches on noises and fluctuating transitions. Let us apply the proposed method to simulate the current fluctuating transitions of the reverse biased semiconductor reference diode. The example of the experimental signal from the diode is given in fig. 3.

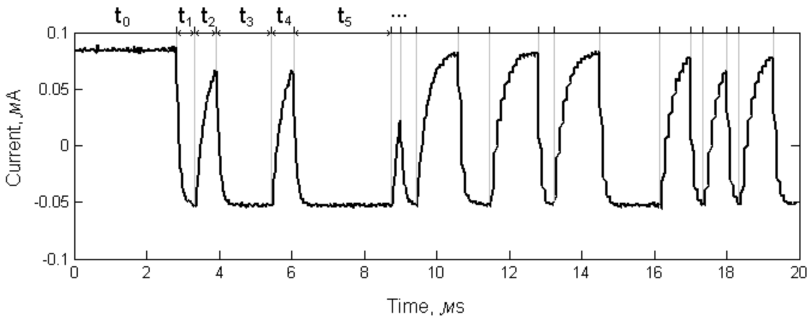
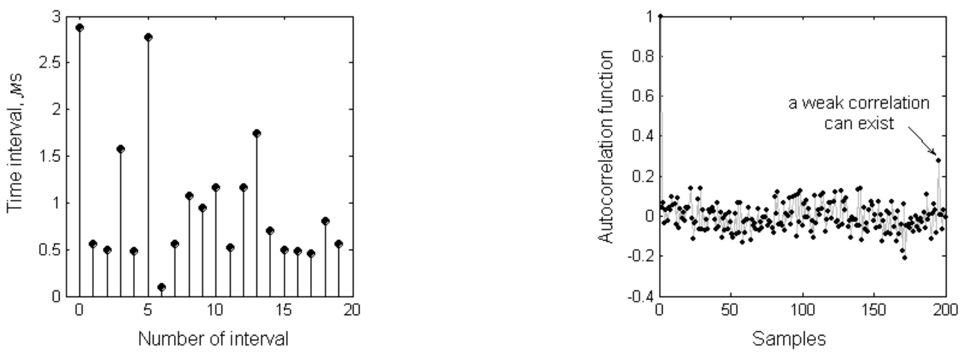


Fig. 3. The experimental signal from a semiconductor noise generator

Pre-processing. It is rather hard to approximate this signal directly. To simplify the situation the signal should be decomposed into its main logical elements. As one of those, the intervals t_0, t_1, t_2, t_3 between transitions can be taken. These periods include the stochastic information and are the main objects of the interest; the signal behaviour in the area between transitions is quite trivial. The presentation of the signal in the discrete-event form is shown in fig. 4*a*. The intervals plotted in this figure correspond to the part of the signal showed in fig. 3 – from 0 up to 20 μs. In the fig. 4*b* a slight correlation can be observed. The approximation of this weakly correlated signal will be the main task of the current section.



a) *b)*

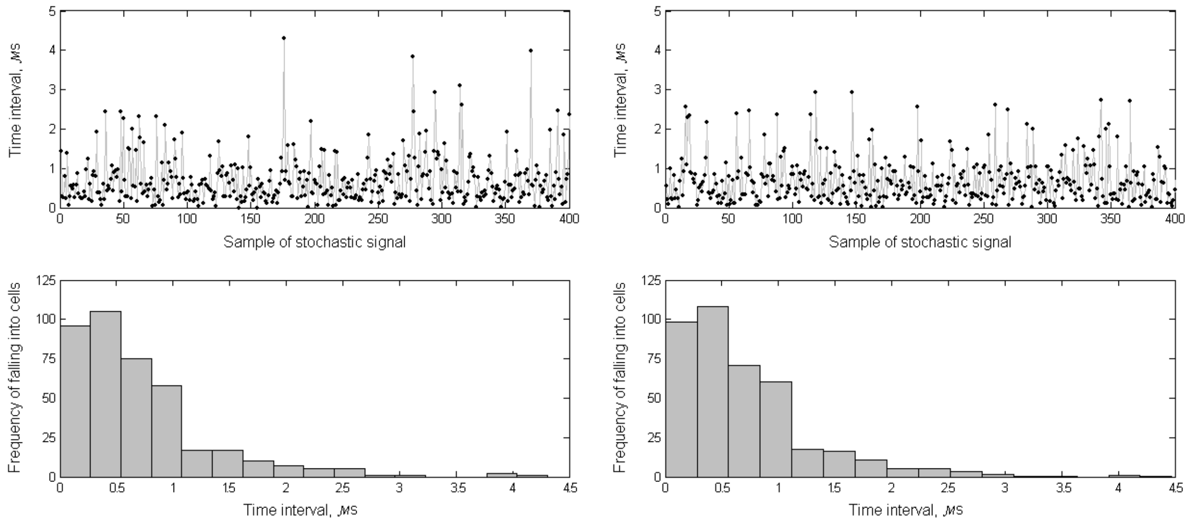
Fig. 4. The result of the pre-processing stage: the discrete-event form of the experimental signal (a) and its autocorrelation function (b)

Simulation. To simulate the produced "interval function" four uniformly distributed random vectors were taken. The structure of the simulating three layer perceptron was 4x4x1 in the 1st, 2nd and 3rd layers respectively. Sigmoid function was used as activating one for the 1st and 2nd layers. The neuron of the 3rd layer was linear.

The statistical parameters of experimental and simulated periods are presented in table 2. The signals themselves can be seen in fig.5.

Table 2. Several statistical characteristics of the experimental and simulated signals

Statistical characteristics	**Training signal**	**Simulated signal**	**Relative deviation**
mean	69.473	69.338	0.2 %
standard deviation	61.020	59.873	1.9 %
minimal value	0.0000	0.0932	–
maximal value	431.00	447.38	3.8 %



a) *b)*

Fig. 5. The experimental (a) and simulated by ANN (b) stochastic signals and their histograms

6. Conclusions

The proposed method showed good results in approximation of a normally distributed random signal. The ANN was successfully applied for simulation of fluctuating transitions of the reverse biased semiconductor reference diode. The slight correlation of the "interval" function did not lead to noticeable problems during ANN simulation. However it should be noted, that the proposed approach is applicable only for δ-correlated stochastic signals. Special transformations (convolution, sum) or ANN with feedback has to be used to simulate other signals. In the second case, more complex ANN could be trained using the same scheme from fig. 2.

To enhance the proposed method, it is intended to implement advanced statistical algorithms for ANN performance calculation during training. The ANN training procedure can be accelerated by application of a parallel computing architecture. Fortunately, both Monte-Carlo and genetic training algorithms can easily be parallelised.

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